

Macroeconomics (Prof. Cazzavillan)

Problem set 4*

Due date: November 24th , 2006

1 Question 1: the Barro model

Consider the Barro model where growth is sustained by the externality generated by the flow of public expenditure and production takes this form¹:

$$y = Ak^\alpha g^{1-\alpha}$$

Given the system characterizing the optimal intertemporal solution

$$\frac{\dot{c}}{c} = \frac{(1-\tau)A^{1/\alpha}\tau^{(1-\alpha)/\alpha} - \rho}{\sigma}$$
$$\dot{k} = (1-\tau)Ak(A\tau)^{(1-\alpha)/\alpha} - c$$

$c \geq 0$, $k \geq 0$, $k_0 > 0$ and known, check whether the transversality condition is satisfied in equilibrium, e.g. find the parameter restrictions that ensure TVC holds.

2 Question 2: Lucas model

Consider the Lucas model where the social planner's utility function is given by:

$$u(c(t)) = \ln c(t)$$

The production function in the sector producing final goods is:

$$Y(t) = AK(t)^\alpha H_1(t)^{1-\alpha}$$

and the sector that produce human capital is characterized by the following linear production function:

$$\dot{H}(t) = \phi H_2(t)$$

Q2.1

Express the two production function in per capita terms.

Q2.2

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¹I am omitting time index, each variable should be written $c(t)$!

Set up the social planner's problem (with the 2 constraints) and find the FOCs. [Hints: h behaves exactly like k , you have 2 multipliers, say λ_k and λ_h and the FOC with respect to h are exactly as the FOC for k , with the respective multiplier.] Just write the FOCs and that's it.

Q2.3

What are the 2 transversality conditions? Why do you need two transversality conditions? Given that $\dot{k}/k = \dot{c}/c = \phi - \rho$, find the parameter restriction that ensure TVCs hold.

Q2.4

Show that output growth is a weighted average of the growth rate of the two sectors.

Q2.5

Which stylized facts can this model explain?

3 Question 3: Ramsey in discrete time

Consider a discrete world where the representative agent has the usual CES utility function. The representative firm is characterized by the a Cobb Douglas production function:

$$Y_t = AK_t^\alpha L_t^{1-\alpha}$$

Population grows at the rate of n and δ is between $(0, 1)$

Q3.1

Set up the representative agent problem and find the Euler's equation (FOCs of the consumer)

Q3.2

Characterize the decentralized solution: how would you define a competitive equilibrium?²

Q3.3

Is there a steady state? What conditions the production function has to satisfy in order to ensure the existence of a steady state? Is it dynamic efficient?

²Also in the discrete time we have a TVC, it is defined exactly as in continuous time but with β^t as discount factor.