Macroeconomics (Prof. Cazzavillan)
Problem set 1*

October 20, 2006

Consider a continuous time neoclassical growth model (Ramsey model). This economy is populated by a number of identical households characterized by the following period CIES (constant intertemporal elasticity of substitution) utility function:

$$u(c(t)) = \frac{c(t)^{1-\theta} - 1}{1-\theta}$$

where $\theta \neq 1$ and $\theta \geq 0$.

There is large number of identical firms that share the same the technology represented by the following production function:

$$Y(t) = AK(t)^{\alpha} L(t)^{1-\alpha}$$

where A is a constant that represent the state of technology, here assumed to be exogenous. There is population growth, $\dot{L}(t)/L(t) = n$ and capital has an instantaneous rate of depreciation equal to δ.

1 Question 1: conditions for the existence of an optimal path

Q1.1. This utility function is also called CRRA, constant relative risk aversion coefficient. What is the relative risk aversion coefficient? How does it relate to the intertemporal elasticity of substitution? Interpret. Q1.2
Verify that the production function is neoclassical (it satisfies the assumptions illustrated in class) and it satisfies the Inada conditions.
Q1.3
Set up and solve the firm maximization problem.
Q1.4
Find the aggregate and the individual budget constraint.
Set up the utility maximization problem of the representative agent over the infinite horizon, using $\rho > 0$ as discount rate ($e^{-\rho t}$ as discount factor). What are the constraints that have to be satisfied?
Q1.5
Set up the Hamiltonian and derive the first order conditions (also called Euler equation) for an optimal path.
Q1.5

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How would you define a competitive equilibrium for this economy? Besides the FOC what are the other conditions required for the definition of an equilibrium?

2 Question 2: transitional dynamics through the phase diagram and linerization around the steady state

Q2.1
Find the steady state values of c and k. Use a phase diagram to represent the two ordinary differential equations describing the optimal path of $c(t)$ and $k(t)$ (Put $k(t)$ on the x axis and $c(t)$ on the y axis, this is just a convention). Use a set of arrows to describe the transitional dynamics. How many steady states are there? Are they stable or not?
Q2.2
Linearize the system of the two differential equations for $c(t)$ and $k(t)$ around the steady state. Find the Jacobian matrix of the first derivatives . Which kind of information do you need to check the stability of the system? Does this hold in general or is it a special case of 2X2 system?

3 Question 3: technological progress

Suppose that the production function exhibits labor-augmenting technological change, $Y(t) = F(K(t), A(t)L(t))$ and that technological change is growing at rate g, $\dot{A}(t)/A(t) = g$. How this change in the production function would affect the results?[Hints: define the production function in terms of per effective unit of labor instead of just per capita, $\hat{k}(t) = K(t)/A(t)L(t)$. Define also c per effective unit of capital instead of per capita ($\hat{c}(t) = C(t)/A(t)L(t)$. You do not need to solve everything again, just make two small adjustments in the FOC.]