Macroeconomics (Prof. Cazzavillan)
Problem set 3*

Due date: November 17 , 2006

1 Question 1: comparing endogenous and neoclassical growth

Assume there are two countries, A and B, characterized by the following production functions:

$$Y_i = A_i K_i^{\alpha} L_i^{1-\alpha} \overline{K_i}^{\psi_i}$$

where $i = A, B$. The two countries differ only for the technology, $A_A > A_B$. For the time being, assume that $\psi_A = \psi_B = \psi$.

Assume the consumption side is characterized by a log utility function,

$$u(c) = ln(c)$$

$Q1.1$
Give some intuition about the utility function.
$Q1.2$
Besides $\psi_A = psi_B = \psi$, assume that $\psi + \alpha < 1$ in both countries. Find the steady state of the two economies and compare them. [Show the calculations for at least 1 country, but the equations describing the equilibrium and the solutions for both countries.]
$Q1.3$
Does the two countries 'converge'to the same steady state? Which type of convergence do you see?[1]Is the more productive country growing more than the other one?
$Q1.4$
Could you think about a policy that would lead to convergence to the same steady state?
$Q1.5$
Now assume that $\psi_A + \alpha = 1$ and $\psi_B + \alpha < 1$ Do you still obtain absolute convergence in growth rate? What could be represented in ψ?Which kind of policy should country B implement in order to follow the growth path of country A [Think about how you could influence they key parameter..]?
$Q1.6$

*TA: Enrica De Cian enrica.decian@unive.it.

[1]Conditional convergence is the convergence of each country to its own steady state whereas absolute convergence is the convergence to the very same long-run growth rates.

Now assume that $\psi_A + \alpha = 1$, $\psi_B + \alpha = 1$, $\psi_B = \psi_A$, always with $A_A > A_B$. Describe the dynamics of one of these two economies. What are the conditions to ensure long-run sustained growth? Find the steady state, if a some sort of steady state exists. Is one of the two country growing faster? How would you design a policy to ensure absolute convergence (in growth rates)?

2 Question 2: Linearization

Consider the following system of equation, describing the intertemporal equilibrium in a Ramsey model, where $\hat{k}(t) = K(t)/A(t)L(t)$ and $\hat{c}(t) = C(t)/A(t)L(t)$.

$$\dot{\hat{k}} = f(\hat{k}) - (g + n + \delta)\hat{k} - c$$

$$\frac{\dot{\hat{c}}}{\hat{c}} = \frac{f(\hat{k})' - (\delta + \rho + \theta g)}{R(c)}$$

Use the following functions:

$$Y(t) = K(t)^{\alpha}[A(t)L(t)]^{1-\alpha}$$

$$u(c(t)) = log(c(t))$$

Q2.1
Find the expression for $f(\hat{k})$ and $R(\hat{c})$.
Q2.2
Linearize the system of equation explicitly using the expression you found in point 2.1 for $f(\hat{k})$ and Q2.3
Describe the dynamics of the system.
Q2.4
Write down the characteristic equation and find the eigenvalues. Check that their sign is in line with the analysis of the determinant.