Macroeconomics (Prof. Cazzavillan)
Problem set 5*

Due date: before the final exam

1 Question 1: Overlapping generation model

Q1.1
Consider the two periods overlapping generation model with CIES utility and Cobb-Dougals production function $Y = AK^{\alpha}L^{1-\alpha}$. There is population growth equal to n. Find the Euler condition describing the optimal rule for consumption.
Q1.2
Find c^*1, c^*2 and s^*.
Q1.3
From the equilibrium condition $s = i$ find the difference equation describing the evolution of k_{t+1} as a function of k_t.
Q1.4
Show that there exist a unique steady state and, when $\sigma \geq 1$, the steady state is stable.
Q1.5
Now specialize the utility function to $\sigma = 1$, e.g. to :

$$lnc_{1t} + \beta lnc_{2t+1}$$

where $\beta = \frac{1}{1+\theta}$. Characterize the equilibrium and derive conditions under which it is dynamically inefficient.
Q1.6
Show that when there is dynamic inefficiency, it is possible to construct a pay-as-you-go social security scheme which creates a Pareto improvement relative to the initial competitive allocation.

2 Question 2 (Optional): Productivity shock in the Ramsey model

Consider the discrete version of the Ramsey model where now production depends on a stochastic productivity term U_t:

$$Y_t = U_t K_t^{\alpha} L_t^{1-\alpha}$$

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Labor is fixed and can be normalized to one. The representative agent now maximizes:

$$E[\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\theta}}{1-\theta} | \Omega_t]$$

s.t.

$$C_t + S_t = U_t K_t^{\alpha} L_t^{1-\alpha}$$

$$K_{t+1} = (1-\delta) K_t + S_t$$

Q2.1
Put the 2 constraints together,assume $\delta = 0$ and find the FOCs at time t with respect to C_t and K_{t+1} [You could for example set up Lagrangian for two subsequent periods]. Q2.2
Interpret the FOCs for different level of σ, e.g. how does σ and r_{t+1} affect the consumption profile (substitution versus smoothing)?
Q2.3
Assume that $U_t = U$ and find the deterministic steady state values for K and C. Use these conditions to give some intuition about the effect of an unexpected permanent increase in U and of an unexpected transitory increase in U on C, S (I) and Y. Does it depends on σ?