# Financial Econometrics

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**What is the financial econometrics?**
Broadly speaking, financial econometrics is to study quantitative problems arising from finance. It uses econometric techniques and economic theory to address a variety of problems from finance. These include building financial models, estimation and inferences of financial models, volatility estimation, risk management, testing financial economics theory, capital asset pricing, derivative pricing, portfolio allocation, risk-adjusted returns, simulating financial systems, hedging strategies, among others.
Technological invention and trade globalization have brought us into a new era of financial markets. Over the last three decades, enormous number of new financial products have been created to meet customers' demands. For example, to reduce the impact of the fluctuations of currency exchange rates on a firm's finance, which makes its profit more predictable and competitive, a multinational corporation may decide to buy the options on the future of foreign exchanges; to reduce the risk of price fluctuations of a commodity (e.g. lumbers, corns, soybeans), a farmer may enter into the future contracts of the commodity; to reduce the risk of weather exposures, amuse parks (too hot or too cold reduces the number of visitors) and energy companies may decide to purchase the financial derivatives based on the temperature. An important milestone is that in the year 1973, the world's first options exchange opened in Chicago. At the very same year, Black and Scholes (1973) published their famous paper on option pricing and Merton (1973a) launched general equilibrium model for security pricing, two landmarks for modern asset pricing. Since then, the derivative markets have experienced extraordinary growth. Professionals in finance now routinely use sophisticated statistical techniques and modern computation power in portfolio management, securities regulation, proprietary trading, financial consulting and risk management.
Financial econometrics is an active field of integration of finance, economics, probability, statis- tics, econometrics and applied mathematics. Financial activities generate many new problems, economics provides useful theoretical foundation and guidance, and quantitative methods such as statistics, probability and applied mathematics are essential tools to solve quantitative problems in finance.
There are several books on financial econometrics and related areas. Campbell et al.(1997) is an excellent book on a comprehensive overview of financial econometrics. A distinguished feature of the book is that it includes many empirical studies. Gouriéroux and Jasiak (2001) give a concise account on financial econometrics, but some prerequisites are needed. Tsay (2002) is an excellent book on the analysis of time series. It emphasizes on the methodological power of time series techniques on the analysis of financial data.

While the programme is expected to be revised according to students' background and interest, I listed the topics covered in my class to give readers an overview.

1. Overview of statistical methods

2. Predictability of asset returns

3. Discrete time volatility models: GARCH and Markov Switching models

4. Efficient portfolios and CAPM

5. Multifactor pricing models

6. Simulation methods for financial derivatives

7. Other selected topics:

Econometrics of financial derivatives*
Forecast and management of market risks*
Multivariate time series in finance*
Systemic risk*

* These topics will be discussed according to participants interests and background.

The field of financial econometrics is much wider than what I presented here. However, the above topics give students a sample of taste on the subject.

**Prerequisites**
Students are expected to be comfortable with statistics, econometrics at the level of first-year courses and basics financial economics.

**Examination policy**
During the course, students are asked to give at least one short presentations of a paper. At the end of the course, they must complete a short paper either in the form of a critical essay (akin to a "referee report"), choosing from a list of recent working papers, or in the form of an applied project, using the tools discussed during the course. The grading will be based on the presentation and class participation (40%), and on the short paper (60%).

**More on the programme**

***Asset pricing and CAPM***

Asset pricing tries to understand the prices of claims with uncertain payments. Stock holders, for example, are entitled to the dividend payments over the life time of the stock, which are uncertain. The derivative pricing in the last section is based on relative pricing, inferring the derivative value given the prices of some other assets such as the stocks and the riskfree bond. We do not ask where the price of other assets came. There is also a huge literature on "absolute pricing", valuating each asset by reference to its exposure to fundamental sources of macroeconomic risk. An excellent book on this topic is Cochrane (2001).

The quantification of the tradeoff between risk and expected return is one of fundamental problems in financial econometrics. While common sense suggests that riskier investments will generally yield higher returns, it is not until the birth of the Capital Asset Pricing Model (CAPM) that economists were able to quantify risk and the reward for baring it. Markowitz (1959) laid the groundwork for the CAPM and postulated the investor's portfolio selection problem in terms of the expected return and variance of the return. He argued that investors would optimally hold a mean- variance efficient portfolio. Sharpe (1964) and Lintner (1965) developed further the Markowitz's work and showed that market portfolio (e.g. S&P 500 index, as a proxy) is a mean-variance efficient portfolio. As a consequence of this, they showed the following CAPM: The expected excessive return of any asset over a risk free bond is a multiple, called market $\beta$, of the excessive return of the market portfolio. The market $\beta$ measures the risk of the asset relative to the market portfolio. The CAPM quantifies exactly how the expected return depends on the risk of the asset, measured by the market $\beta$.

Since its publication, various statistical techniques have been developed to verify the validity of the CAPM in empirical finance. The early evidence was largely positive. Yet, in the late 1970's, some evidence against the CAPM began to appear in the so-called anomalies literature in which firms can be grouped according to their characteristics to form a portfolio that can be more efficient than the market portfolio. While the evidences against the CAPM are still controversial, various extensions of the CAPM have been proposed to better capture the market risks. These include intertemporal CAPM (Merton, 1973b), multifactor pricing model such as the Arbitrage Pricing Theory (Ross, 1976), and consumptions based CAPM, among others. These models can be more generally represented by the stochastic discount factor model. A different CAPM amounts to choose a different stochastic discount factor (see, e.g., Cochrane, 2001). They form spectacularly beautiful theory on asset pricing.

Testing the validity of various versions of CAPMs attracts a lot attention in empirical finance. Various testing procedures and statistical methods have been proposed and studied. Statistical techniques have also been used to select risk factors that explain the expected returns of assets over a period of time. For example, Fama and French (1993) build a famous three-factor CAPM to explain the expected excessive returns of assets. They test CAPM using the following three factors: the CRSP value-weighted stock index (a proxy of the market portfolio), the difference of returns on a portfolio of low and high market value of equity firms, the difference of returns on a portfolio of high and low book-to-market value firms. Sophisticated statistical models have been introduced to model the behaviors of consumptions and habits and advanced statistical methods have been applied to test the consistency of these models with empirical financial data.

### *Stochastic modeling and statistical inferences*

The valuation of financial derivatives depends largely on the stochastic model assumptions on the price dynamics of underlying assets. A different asset class requires a different class of stochastic models. For example, the geometric Brownian motion, which has a constant rate of expected return and volatility, cannot be used to model bond yields, which possess the heteroscedasticity and mean- revision feature. As they rise, the interest rates tend to be more volatile and there is a positive pulling the rates down when they exceed a mean level, while as the interest rates go down, the volatility tends to be smaller and there is a positive force driving the rates up when they are below the mean reversion level. Managing and modeling the risks such as natural disaster and weather require a very different class of stochastic models. A stochastic model can only capture certain aspect of underlying stochastic dynamics. This is why there are many models being introduced for the price dynamics of various asset classes.

Options prices depend on the underlying parameters of stochastic processes. The question then arises how to efficiently estimate the parameters from a discretely observed stochastic diffusion process. For an overview, see Fan (2003). If the model has been parameterized, then the maximum likelihood method is a natural candidate. However, except for a few specific models, the likelihood function is difficult to derive analytically and hence hard to implement. One possible remedy is to use the generalized method of moments (Hansen, 1982) to derive some estimation equations and some other features such as local time of stochastic processes to derive a different set of equations. Other methods involve using approximate likelihood, resulting from the Euler approximation or higher-order approximations of stochastic processes when the sampling interval is small. This is more feasible nowadays, thanks to the availability of high frequency data. The biases of an approximated likelihood method can be reduced by using some calibration methods such as the indirect inference by Gourieroux et al. (1993).

Nonparametric models arise naturally in financial modeling. They aim at reducing modeling biases of parametric models and validating their goodness of fit to financial data. Many of such parametric models are simple and convenient ones to facilitate mathematical derivations and statistical inferences. They are not derived from economics theory and cannot be expected to fit all financial data. While the asset pricing theory gives nice pricing formulas when the underlying price dynamic is correctly specified, it offers litter guidance in choosing or validating a model. Hence, there are genuine needs for flexible stochastic modeling, and nonparametric methods offer a unified and elegant treatment.

Nonparametric approaches have recently been introduced to estimate return, volatility, transition densities and state price densities of stock prices and bond yields. See Fan (2003). They are also useful for examining the extent to which the dynamics of stock prices and bond yields vary over time. They have immediate applications to the valuation of bond price and stock options and management of market risks. They can also be employed to test economic theory such as the CAPM and other stochastic discount models, and answer the questions such as if the geometric Brownian motion fits certain stock indices, whether the Cox-Ingersoll-Ross model fits yields of bonds, and if interest rates dynamics evolve with time. In these testing problems, nonparametric models serve as natural alternative models to the null hypotheses. Furthermore, based on empirical data, one can also fit directly the observed option prices with their associated characteristics and checks if the option prices are consistent with the theoretical pricing formula. They can also be used to testing whether an underlying asset follows a time-homogeneous Markovian process and can even be used as estimation tools for parametric models.

### *Volatility, portfolio optimization and risk management*

Volatility pervades almost every facet of financial econometrics. It is used in pricing financial derivatives, portfolio allocation to control and manage risks, and computation of risk-adjusted returns for comparisons of relative performance of various financial investments (e.g. mutual funds). It measures the risk of a portfolio and is associated with capital requirement in banking regulations. The topic is prominently featured at the heart of the financial econometrics.

There are two popular classes of widely used models for volatility of discretely observed time series. One is the ARCH and GARCH models and their various extensions. For an overview of the subject, see Engle (1995). These models attempt to capture several important stylized features in financial markets. These include volatilities clustering, heavy tail and asymmetric distributions of returns, persistence of autocorrelation in absolute or squared returns, and leverage effect (stock price movement is negatively correlated with the change of volatility). Various statistical procedures, including quasi-maximum likelihood estimator and robust methods, have been introduced to fit the ARCH-GARCH models. Various efforts have been devoted to investigate the property of this class of models and the performance of statistical procedures. Modeling volatility matrices of multivariate time series poses many new statistical problems with new challenges, as the number of parameters grows quickly with the number of the assets. Recent arrival of high frequency data give rises to new interesting statistical problems.

Stochastic volatility models are another important class for capturing the stylized features of volatilities in financial data. The challenge is that the volatility is not directly observable. Instead, it is driven by a different unobservable random process. The problem here is similar to the error- in-variable regression in statistics (see, e.g., Carroll, Ruppert and Stefanski, 1995). There is a large literature on studying the statistical issues and probabilistic aspects of the models. See Shephard (2004) for an overview.

The portfolio allocation can be formed based on the mean and variance consideration in a similar way to the fundamental work of Markowitz (1959), Sharpe (1964) and Lintner (1965). Its basic idea is to maximize the expected returns while controlling the risks. It can be formulated as a constrained optimization problem. The expected return and volatility play a prominent role in the portfolio allocation, and statistical techniques are widely used for modeling the returns, volatilities as well as the risk management.

Risk management is to identify risk sources, to measure risks and to control and manage risks. There are large efforts in statistical community on defining and forecasting risk measures. They are directly related to the volatilities of asset returns. and have been widely used in security and bank regulations, proprietary trading, and risk managements. Statistical methods have been widely used in such an endeavor. For an overview, see Jorion (2000).

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